

| $\alpha$          | Cuadrante  | $\cos \alpha$         | $\sin \alpha$         | $\tan \alpha$         |
|-------------------|------------|-----------------------|-----------------------|-----------------------|
| 0                 |            | 1                     | 0                     | 0                     |
| $\frac{\pi}{6}$   | <i>I</i>   | $\frac{\sqrt{3}}{2}$  | $\frac{1}{2}$         | $\frac{\sqrt{3}}{3}$  |
| $\frac{\pi}{4}$   | <i>I</i>   | $\frac{\sqrt{2}}{2}$  | $\frac{\sqrt{2}}{2}$  | 1                     |
| $\frac{\pi}{3}$   | <i>I</i>   | $\frac{1}{2}$         | $\frac{\sqrt{3}}{2}$  | $\sqrt{3}$            |
| $\frac{\pi}{2}$   |            | 0                     | 1                     | No existe             |
| $\frac{2\pi}{3}$  | <i>II</i>  | $-\frac{1}{2}$        | $\frac{\sqrt{3}}{2}$  | $-\sqrt{3}$           |
| $\frac{3\pi}{4}$  | <i>II</i>  | $-\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$  | -1                    |
| $\frac{5\pi}{6}$  | <i>II</i>  | $-\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$         | $-\frac{\sqrt{3}}{3}$ |
| $\pi$             |            | -1                    | 0                     | 0                     |
| $\frac{7\pi}{6}$  | <i>III</i> | $-\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$        | $-\frac{\sqrt{3}}{3}$ |
| $\frac{5\pi}{4}$  | <i>III</i> | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{2}}{2}$ | 1                     |
| $\frac{4\pi}{3}$  | <i>III</i> | $-\frac{1}{2}$        | $-\frac{\sqrt{3}}{2}$ | $\sqrt{3}$            |
| $\frac{3\pi}{2}$  |            | 0                     | -1                    | No existe             |
| $\frac{5\pi}{3}$  | <i>IV</i>  | $\frac{1}{2}$         | $-\frac{\sqrt{3}}{2}$ | $-\sqrt{3}$           |
| $\frac{7\pi}{4}$  | <i>IV</i>  | $\frac{\sqrt{2}}{2}$  | $-\frac{\sqrt{2}}{2}$ | -1                    |
| $\frac{11\pi}{6}$ | <i>IV</i>  | $\frac{\sqrt{3}}{2}$  | $-\frac{1}{2}$        | $-\frac{\sqrt{3}}{3}$ |
| $2\pi$            |            | 1                     | 0                     | 0                     |

## 1. Identidades básicas de las funciones trigonométricas

$$a) \tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{1}{\tan x}, \quad \sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}$$

$$b) (\sin x)^2 + (\cos x)^2 = \sin^2 x + \cos^2 x = 1$$

$$c) 1 + \tan^2 x = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$$

## 2. Signos de las funciones trigonométricas.

*Cuadrante I:* Todas la FT son positivas. *Cuadrante II:*  $\sin$  y  $\csc$  positivas, el resto negativas.

*Cuadrante III:*  $\tan$  y  $\cot$  positivas, el resto negativas. *Cuadrante IV:*  $\cos$  y  $\sec$  positivas, el resto negativas.

3. Fórmulas de Reducción. Usaremos FT para designar cualquiera de las 6 funciones trigonométricas y coFT su respectiva cofunción (la cofunción del  $\sin$  es  $\cos$ , de la  $\tan$  es  $\cot$  y de la  $\sec$  es la  $\csc$ ).  $0 < \alpha < \frac{\pi}{2}$ .

$$a) \text{FT}\left(\frac{\pi}{2} \pm \alpha\right) = \pm \text{coFT}(\alpha) \quad b) \text{FT}\left(\frac{3\pi}{2} \pm \alpha\right) = \pm \text{coFT}(\alpha)$$

$$c) \text{FT}(\pi \pm \alpha) = \pm \text{FT}(\alpha) \quad d) \text{FT}(2\pi \pm \alpha) = \pm \text{FT}(\alpha)$$

*Observación:* El signo depende del cuadrante donde este situado  $\frac{\pi}{2} \pm \alpha$ ,  $\frac{3\pi}{2} \pm \alpha$ ,  $\pi \pm \alpha$

4. Fórmulas para funciones trigonométricas para  $\alpha \pm \beta$ 

$$a) \sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \quad b) \cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$c) \tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

$$d) \sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$e) \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$f) \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

5. Fórmulas para  $\alpha/2$ .

$$a) \sin^2 \frac{\alpha}{2} = \frac{1 - \cos \alpha}{2} \quad b) \cos^2 \frac{\alpha}{2} = \frac{1 + \cos \alpha}{2} \quad c) \tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$$

6. Productos de  $\sin$  y  $\cos$ 

$$a) \sin \alpha \cdot \cos \beta = \frac{1}{2} \sin(\alpha - \beta) + \frac{1}{2} \sin(\alpha + \beta)$$

$$b) \cos \alpha \cdot \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta)$$

$$c) \sin \alpha \cdot \sin \beta = \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \cos(\alpha + \beta)$$

7. Suma y Diferencia de  $\sin$  y  $\cos$ 

$$a) \sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2} \quad b) \sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

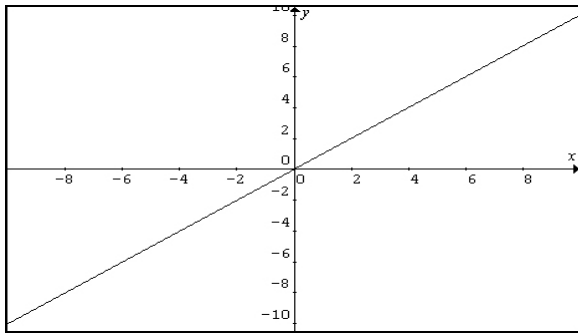
$$c) \cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2} \quad d) \cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}$$

8. Teoremas de seno y del coseno. En un triángulo cualquiera de ángulos interiores  $\alpha$ ,  $\beta$  y  $\gamma$  y lados opuestos  $a$ ,  $b$  y  $c$ , respectivamente, se tiene que:

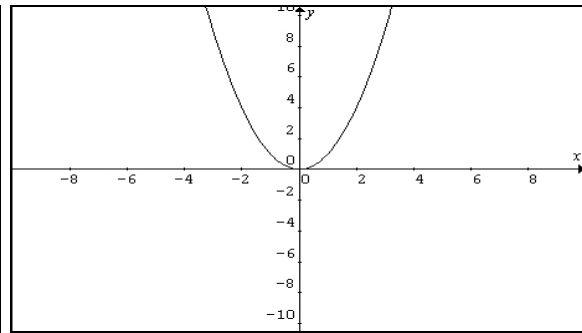
$$a) \text{ Teorema del seno: } \frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

b) Teorema de los cosenos:

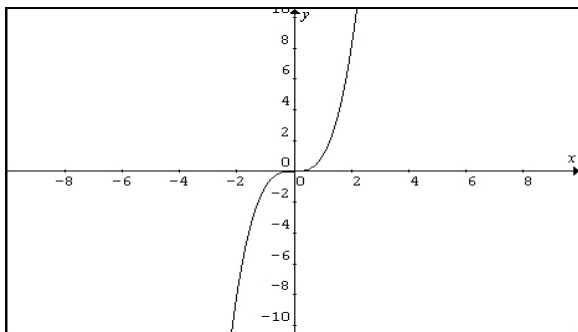
$$i) c^2 = a^2 + b^2 - 2ab \cos \gamma \quad ii) b^2 = a^2 + c^2 - 2ac \cos \beta \quad iii) a^2 = b^2 + c^2 - 2bc \cos \alpha$$



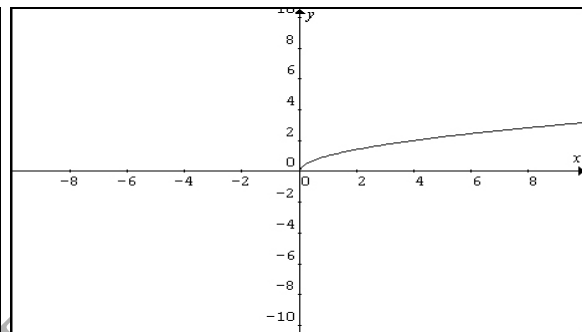
$$y = f(x) = x$$



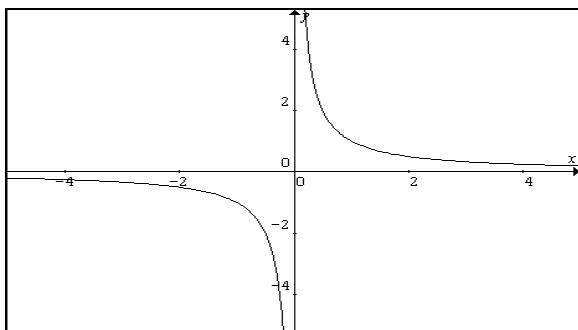
$$y = f(x) = x^2$$



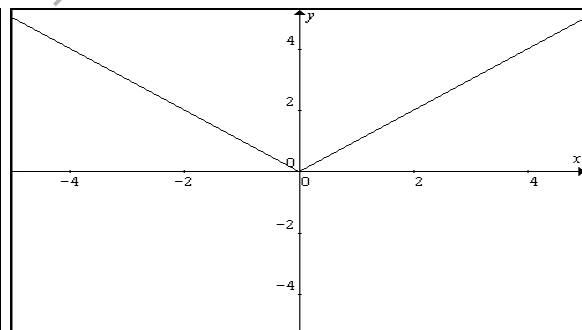
$$y = f(x) = x^3$$



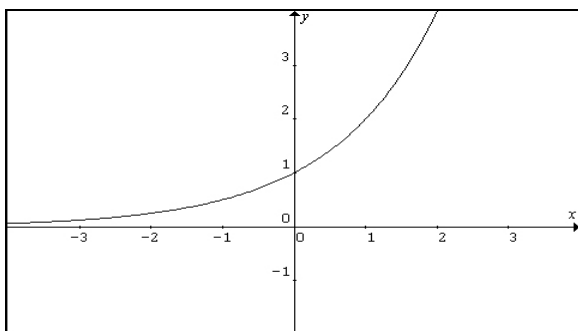
$$y = f(x) = \sqrt{x}$$



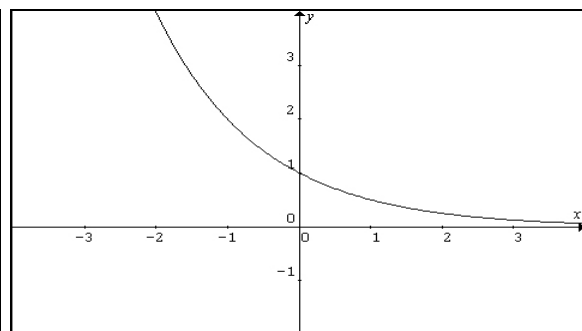
$$y = f(x) = 1/x$$



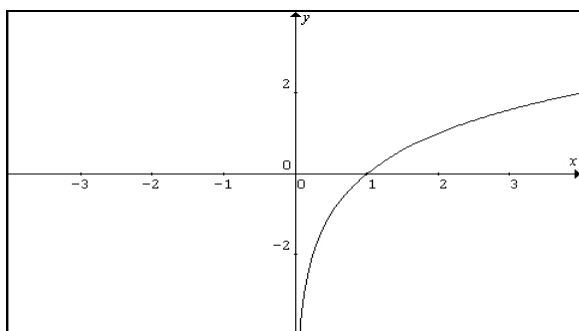
$$y = f(x) = |x|$$



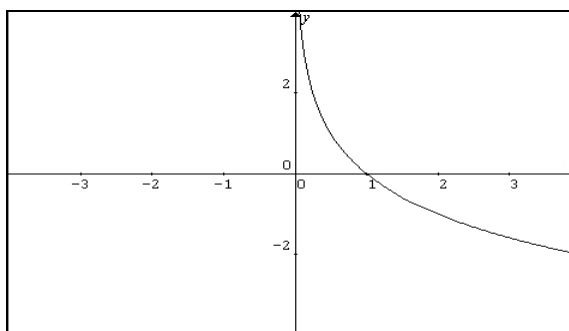
$$y = f(x) = a^x, a > 1$$



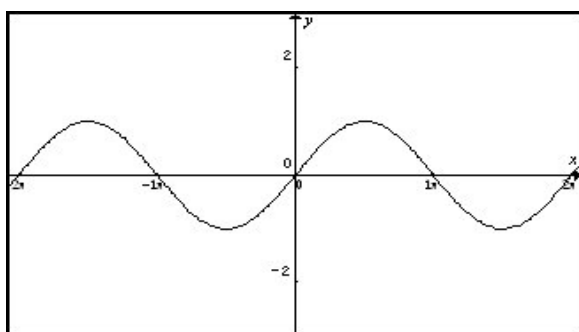
$$y = f(x) = a^x, 0 < a < 1$$



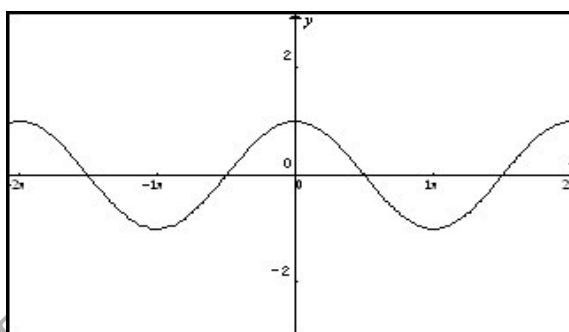
$$y = f(x) = \log_a x, a > 1$$



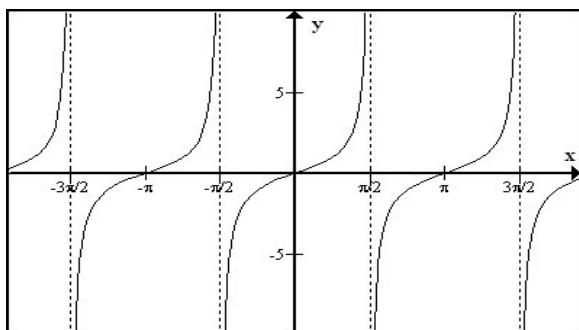
$$y = f(x) = \log_a x, 0 < a < 1$$



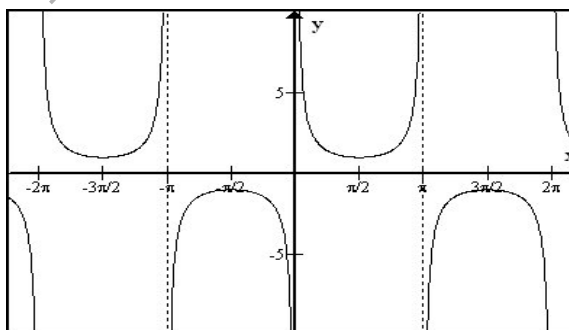
$$y = f(x) = \sin x$$



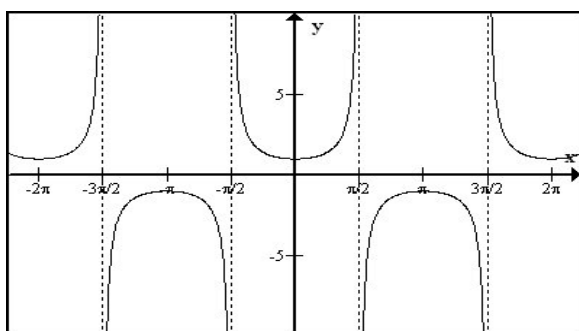
$$y = f(x) = \cos x$$



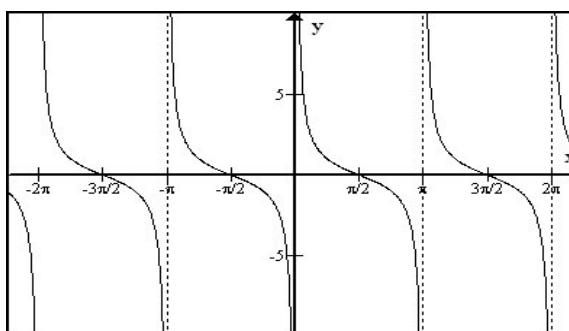
$$y = f(x) = \tan x$$



$$y = f(x) = \csc x$$



$$y = f(x) = \sec x$$



$$y = f(x) = \cot x$$